Effects of Design Parameters on Aerodynamic Performance of a Counter-Rotating Wind Turbine

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The paper investigates effects of design parameters on aerodynamic performance of a counter-rotating wind turbine. The counter-rotating wind turbine has two rotors rotating opposite direction at the same axis. It has been proposed on the basis of the theory which states that a configuration of two rotors having the same swept area at the same axis has higher optimum power coefficient than a conventional configuration of wind turbine having a single rotor. Due to a complex phenomenon arising from aerodynamic interaction between two rotors of the counter-rotating wind turbine, design parameters increase compared to those of a single rotor and influences of these parameters is not yet fully understood. In this study, a blade element momentum theory (BEMT) for a counter-rotating wind turbine is developed to investigate effects of design parameters on aerodynamic performance of a counter-rotating wind turbine. By using the newly-developed method, aerodynamic characteristics of a counter-rotating wind turbine are investigated with design parameters such as combinations of pitch angles, rotating speeds and radii of two rotors.

wind turbine, counter-rotating, blade element momentum theory, power coefficient, tip speed ratio

1   Introduction

A wind resource is one of the clean energy resources and has vast potential that covers more than two hundred times of annual world energy consumption [1]. It is very important to use this wind resource as an energy source for a reduction of fossil fuel dependency and sustainable development. To use the wind resource, a wind turbine that converts wind energy to mechanical energy through rotation of a rotor is used. A worldwide installed capacity of wind turbines has shown a high growth rate because power generation of a wind tubine has lower cost of generation and higher technology maturation than that of other renewable energy resources [2].

In order to reduce cost of wind energy further and maintain continued growth of wind power, it is required to improve the energy conversion efficiency of a wind turbine. The energy conversion efficiency of a wind turbine is usually characterized by its power coefficient that is the ratio of the power extraction from a wind to the power available in the wind. Based on the classical momentum theory, the maximum power coefficient of a wind turbine having an ideal single rotor without any losses is about 59% that is known as the Betz limit [3]. In practice, it is found that the maximum power coefficient of conventional horizontal axis wind turbines having a single rotor is about 40 to 50% due to some losses such as viscous loss, three-dimensional loss, and transmission loss. Over the past few decades, many different concepts and blade designs of a wind turbine have been proposed to improve the maximum power coefficient

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[4].

A couter-rotating wind turbine having two rotors rotating opposite direction at the same axis has been proposed as a new concept to enhance the maximum power coefficient of a wind turbine. Using the classical momentum theory, Newman found that the maximum power coefficient of a wind turbine having two rotors without any losses was increased to about 64% [5]. Recently, based on this result, many researches for a counter-rotating wind turbine have been carried out to obtain more power from a wind than a conventioanl wind turbine having a single rotor [6-11].

Despite these previous researches, it is still difficult to design for aerodynamic optimization of rotors to obtain the maximum power coefficient. That is because there are complex phenomena induced by aerodynamic interactions of two rotors of a counter-rotating wind turbine, unlike a conventional wind turbine having a single rotor. Due to the interactions of two rotors, design parameters like differences of pitch angles, rotational speeds and radii of two rotors are added to design parameters for a single rotor to improve aerodynamic performance of a counter-rotating wind turbine. In addition, effects of these design parameters on aerodynamic performace of a counter-rotating wind turbine are still not fully understood, so that makes the design more complicated. Therefore, as a preliminary study to obtain the optimized design to maximize the power coefficient of a counter-rotating wind turbine and compare it with that of a conventional wind turbine having a single rotor, it is required to investigate the effects of design parameters on aerodynamic performance of a counter-rotating wind turbine.

In this paper, the effects of design parameters on aerodynamic performance of a counter-rotating wind turbine are investigated. The design parameters used in this study are combinations of pitch angles, rotating speeds and radii of two rotors of a counter-rotating wind turbine. For the parametric study, a blade element momentum theory (BEMT) for a counter-rotating wind turbine is developed. By using the newly-developed method, power coefficient characteristics of a counter-rotating wind turbine with variations of design parameters are investigated.

2   Numerical method

The blade element momentum theory (BEMT) is a hybrid rotor analysis method that combines the momentum theory and blade element theory. The momentum theory is a control volume analysis based on the conservation of linear and angular momentum and the blade element theory is an analysis of forces determined solely by the lift and drag characteristics of the airfoil shape of the blade sections by using the assumption that there is no aerodynamic interaction between section elements. The BEMT allows for the inflow on each blade section to be solved for based on

equating of forces from the two theories [12]. The main advantage of the BEMT is that it is a reliable and effective model for rotor design because it is based on solid physical principles and has a remarkably low computing cost.

The flow model for a counter-rotating wind turbine is shown in Fig. 1. It is assumed that the rear rotor operates inside the fully developed stream tube of the front rotor. The velocity fully reduced by the front rotor in the stream tube acts as an inflow for the rear rotor. It is also assumed that the wake from the front rotor induces flow interference into the rear rotor, but the wake from the rear rotor does not affect the flow of the front rotor. Such assumptions are reasonable in most cases except very closely spaced rotors [13].
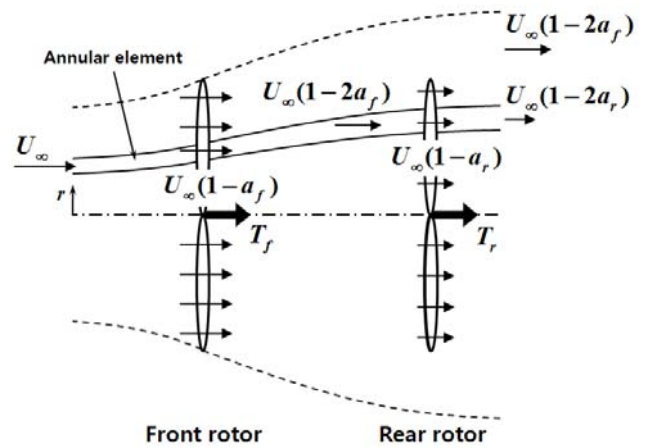
Figure 1 Flow model for the BEMT analysis of a counter-rotating wind turbine with the rear rotor operating inside the stream tube of the front rotor

First, for the front rotor, the axial momentum conservation and the Bernoulli equation are applied on the rotor annulus, and then the incremental thrust is

$$dT_f = \rho dA (U_\infty^2 - U_{far}^2)/2 \tag{1}$$

where ρ is the air density, dA is the rotor annulus area, U is the air velocity and the subscripts f, ∞ and far indicate the front rotor, the free stream and the far downstream velocity, respectively. If the axial induction factor, a_f is defined as the fractional decrease in wind velocity, then the incremental thrust can be written as

$$dT_f = 4\rho U_\infty^2 a_f (1 - a_f) \pi r dr \tag{2}$$

Applying the angular momentum conservation on the front rotor disk and using the angular induction factor, a_f' that is defined as the ratio of the angular velocity at the rotor induced by wake rotation to the rotational velocity of the rotor, Ω_f, the torque exerted on the rotor annulus is

$$dQ_f = 4a'_f(1-a_f)\rho U_\infty \pi r^3 \Omega_f dr \qquad (3)$$

The three-dimensional effect in the blade root and tip region is accounted by using the Prandtl's tip-loss function [12]. This effect is expressed in terms of the Prandtl's correction factor, F, as

$$F = F_{root} \times F_{tip}$$
$$= \frac{2}{\pi}\cos^{-1}(\exp(-f_{root})) \times \frac{2}{\pi}\cos^{-1}(\exp(-f_{tip})) \qquad (4)$$

where f_{root} and f_{tip} are given by

$$f_{root} = \frac{N_b}{2}(\frac{r/R - r_{hub}/R}{(r/R)\sin\varphi}) \qquad (5)$$

and

$$f_{tip} = \frac{N_b}{2}(\frac{1 - r/R}{(r/R)\sin\varphi}) \qquad (6)$$

where N_b is the number of blades, R is the radius of the rotor and φ is the angle of relative wind defined by geometric relations as

$$\varphi = \tan^{-1}[\frac{U_\infty(1-a_f)}{\Omega r(1+a'_f)}] \qquad (7)$$

The Prandtl's correction factor is introduced to the forces derived above, and then the incremental thrust and torque are modified to

$$dT_f = 4F\rho U_\infty^2 a_f(1-a_f)\pi r dr , \qquad (7)$$

and

$$dQ_f = 4Fa'_f(1-a_f)\rho U_\infty \pi r^3 \Omega_f dr \qquad (8)$$

Now, using the blade element theory, the incremental thrust and torque on the anulus area of the rotor disk is

$$dT_f = dF_{N,f} = \frac{1}{2}N_b\rho U_{rel}^2(c_l\cos\varphi + c_d\sin\varphi)cdr$$
$$= \sigma'\pi\rho\frac{U_\infty^2(1-a_f)^2}{\sin^2\varphi}(c_l\cos\varphi + c_d\sin\varphi)rdr \qquad (9)$$

and

$$dQ_f = N_b r dF_{T,f} = \frac{1}{2}N_b\rho U_{rel}^2(c_l\sin\varphi - c_d\cos\varphi)crdr$$
$$= \sigma'\pi\rho\frac{U_\infty^2(1-a_f)^2}{\sin^2\varphi}(c_l\sin\varphi - c_d\cos\varphi)r^2dr \qquad (10)$$

where σ' is the local solidity, and it is defined as

$$\sigma' = N_b c / 2\pi r \qquad (11)$$

By equating the thrust equations and torque equations from the momentum theory and blade element theory, the inflow conditions that are expressed by a_f and a_f' on each blade section are determined. Because the equations can not be solved directly, the solution is obtained by an interative numerical approach. Convergence is rapid and is obtained at three or four iterations in most cases.

Once the inflow solution has been obtained, the total thrust and torque are calculated by integrating the equations

$$T_f = \int dT_f = \int_{r_{hub}}^R 4F\rho U_\infty^2 a_f(1-a_f)\pi r dr \qquad (12)$$

and

$$Q_f = \int dQ_f = \int_{r_{hub}}^R 4Fa'_f(1-a_f)\rho U_\infty \pi r^3 \Omega_f dr \qquad (13)$$

Once the total torque is obtained, the power coefficient of the front rotor is determined by

$$C_{P,f} = \frac{\Omega_f Q_f}{\rho\pi R^2 U^3 / 2} \qquad (14)$$

In order to simulate the three-dimensional stall delay effects, the stall delay model developed by Du and Selig [14] is applied to the two-dimensional aerodynamic coefficients. In addition, for the turbulent wake state in which a_f is lager than 0.5 and the thrust determined by momentum theory is invalid, the empirical relationship of the axial induction factor and the thrust coefficient developed by Glauert [15] is applied to the solution determined by the BEMT.

The same mathematical principles are applied to the analysis of the rear rotor of the counter-rotating wind turbine. However, it is assumed that the rear rotor operates inside the fully developed stream tube of the front rotor. The inflow velocity on the plane of the rear rotor is determined by the axial induction factor of the front rotor as follows.

$$U_{\infty,r} = U_\infty(1-2a_f) \qquad (15)$$

The radial expansion of the fully developed stream tube of the front rotor, R_e, is determined by the mass conservation for the rotor plane and far down stream of the front rotor. After some algebraic manupulation, the radial expansion is derived to

$$R_e = R_f\sqrt{\frac{1-a_f}{1-2a_f}} \qquad (16)$$

If there are some positions of the rear rotor outside the radial expansion of the fully developed stream tube of the front rotor due to the difference of the rotor sizes, the inflow velocity on those positions that are unaffected by the front rotor is set to be the free stream wind velocity.
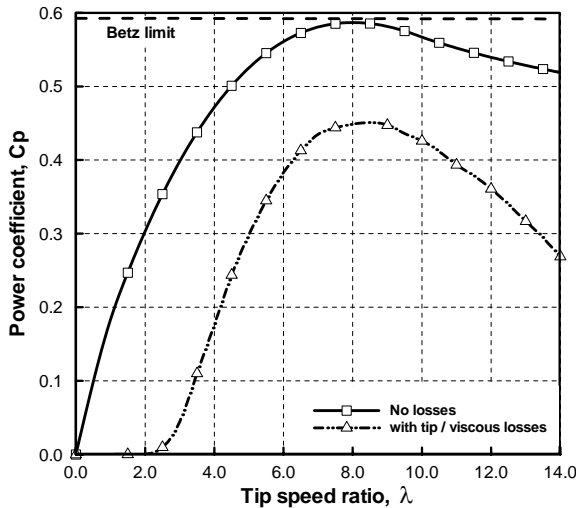
3 Results

A parametric study for a counter-rotating wind turbine is performed by using the BEMT which is developed on the

basis of solid physical principles and reasonable assumptions. To consider characteristics of a counter-rotating wind turbine having two rotors, combinations of pitch angles, rotating speeds and radii of two rotors are chosen as design parameters used in this study.

The baseline rotor used for the parametric study has relatively simple blade geometry to facilitate comparisons by variations of the design parameters. The rotor has three rectangular blades of which chord length is constant with spanwise positions. The rotor solidity is 0.05 and naca0012 airfoil is used for the blade section. For an efficient operation of the rotor, the blades have ideal blade twist, that is $\theta_{twist}(r)=\theta_{tip}/r$ where θ_{tip} is the tip pitch angle. Then the blade sectional pitch angle is defined as $\theta(r)=\theta_0+\theta_{twist}(r)$ where θ_0 is the blade pitch angle. The tip speed ratio (TSR) is defined as the ratio of the blade tip speed to the free stream wind speed and TSR at which a maximum power coefficient is obtained is set to 8.0 which is one of the values in the range of the conventional design TSR. For this design TSR, θ_0 and θ_{tip} is set to 0.0 and 0.3 degree, respectively.

Performance results from the BEMT for the baseline single rotor is shown in Fig. 2. The power coefficient at the design TSR of the rotor in an ideal condition without any losses reached the Betz limit. However, in the case with tip loss and viscous losses like drag and stall, the maximum power coefficient is reduced to 0.458. In the follwing parametric study for a counter-rotating wind turbine, the baseline single rotor with losses is used for the front and rear rotors.

above is used for each rotor of the counter-rotating wind turbine. The rotating speed of each rotor is fixed to be equal each other, and that means TSR of each rotor is fixed to the design TSR, 8.0.

The variation of power coefficient versus pitch differences between two rotors as a set of curves for increasing values of the front rotor pitch is shown in Fig. 3. The maximum power coefficient is obtained when the front rotor pitch is 3.0 degree, not 0.0 degree at which the maximum power coefficient of the baseline rotor is obtained. As the front rotor pitch increases, the operation condition of the front rotor moves away from the design condition of the baseline rotor, and then the power coefficient of the front rotor decreases. In contrast, because interference of the front rotor on the rear rotor is reduced, the power coefficient of the rear rotor increases. For that reason, the maximum power coefficient is obtained when each rotor shares properly the total power, not when the front rotor extracts the maximum power from the wind. In addition, because the interference of the front rotor induces the decrease of inflow velocity on the rear rotor, it is necessary for the pitch of the rear rotor to be smaller than that of the front rotor for the maximum power coefficient. In the case of this study, the maximum value is obtained when the pitch angles of the front and rear rotors are 3.0 and 1.0 degree, respectively.
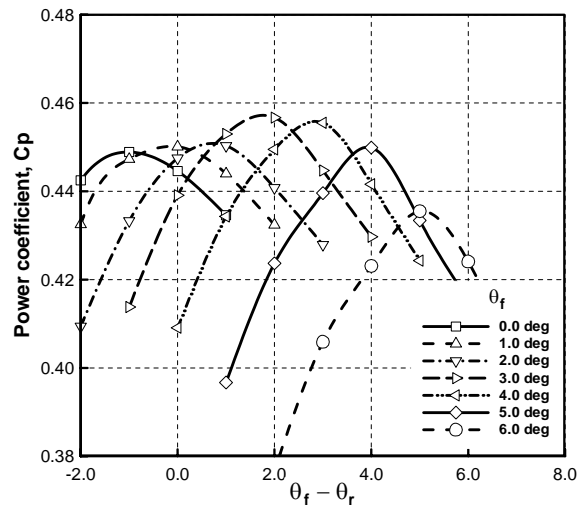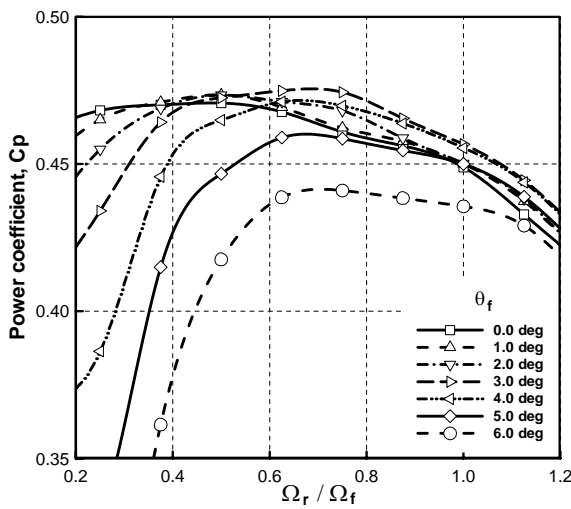

Figure 3   Variation of power coefficient versus pitch differences between two rotors as a set of curves for increasing values of the front rotor pitch ($R_f=R_r$, $\Omega_f=\Omega_r$, $TSR_f=TSR_r=8.0$)

Figure 2   Power coefficient versus tip speed ratio of the baseline single rotor of which the design TSR is 8.0 ($\sigma=0.05$, $\theta_0=0.0$ deg., $\theta_{tip}=0.3$ deg.)

To investigate the effect of combinations of the pitch angles on performance, the same baseline rotor described

Fig. 4 shows the variation of power coefficient versus rotating speed ratio of the two rotors as a set of curves for increasing values of the front rotor pitch. To compare performance by only the rotating speed ratio, the radius of each rotor is fixed to be equal each other, and pitch of the rear rotor and TSR of the front rotor are set to 0.0 degree and 8.0,

respectively.

As shown in Fig. 4, the maximum power coefficient for each pitch setting of the front rotor is obtained below a rotating speed ratio of 1.0 that means two rotors have the same rotating speed. That is because angle of relative wind of the rear rotor is reduced by interference of the front rotor. In the case of having the same rotating speed, due to decrease of inflow velocity on the rear rotor, angle of relative wind of the rear rotor becomes smaller than that of the baseline rotor having maximum performance, and then the power coefficient of the rear rotor decreases. Therefore, the maximum power coefficient is obtained when rotating speed of the rear rotor is reduced to recover the angle of the relative wind for maximum performance of the rear rotor.
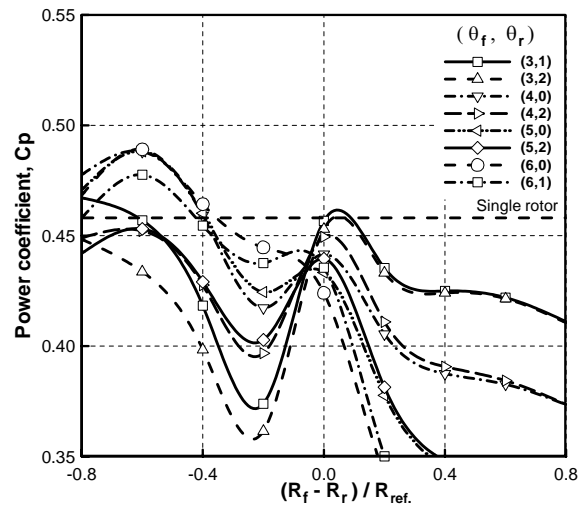

Figure 4 Variation of power coefficient versus rotating speed ratio of the two rotors as a set of curves for increasing values of the front rotor pitch (R_f=R_r, TSR_f =8.0, θ_r=0.0 deg.)

The variation of power coefficient versus radius differences between two rotors with various pitch combinations is shown in Fig. 5. A reference radius to evaluate power coefficient is a bigger radius of two rotors and the radius difference is non-dimensionalized by it. The negative value of a non-dimensional difference of radius means radius of the front rotor is smaller than that of the rear rotor, and it is zero when two radii are equal. The TSR and σ are fixed to be 8.0 and 0.05 with changes of rotating speed and chord length even when rotor radius is changed.

Fig. 5 shows power coefficient decreases when non-dimensional difference of radius moves away from zero except below about -0.2. Because power from the wind is proportional to the area swept by rotor, power coefficient decreases when one of rotor radii decreases. However, power coefficient increases again when a non-dimensional difference of radius decreases over about -0.2, despite de-

crease of the front rotor radius. That is because the outer parts of the rear rotor blades recover the wind velocity as the radial expansion of the fully developed stream tube of the front rotor becomes smaller than the radius of the rear rotor.

Figure 5 Variation of power coefficient versus radius differences between two rotors with various pitch combinations (σ=0.05, TSR_f=TSR_r=8.0)

4 Conclusion

The effects of design parameters on aerodynamic performance of a counter-rotating wind turbine were investigated. For the parametric study, a blade element momentum theory for a counter-rotating wind turbine was established. It was assumed that the rear rotor operated inside the fully developed stream tube of the front rotor. The velocity fully reduced by the front rotor in the stream tube acted as an inflow for the rear rotor.

To consider characteristics of a counter-rotating wind turbine having two rotors, combinations of pitch angles, rotating speeds and radii of two rotors were chosen as design parameters. Through the investigation of the effect of combinations of the pitch angles on performance, it was shown that power coefficient increased when each rotor shared properly the total power, not when the front rotor extracted the maximum power from the wind. It was also shown that power coefficient increased when rotating speed of the rear rotor was reduced to recover the angle of the relative wind for maximum performance of the rear rotor. In addition, it was found that power coefficient increased when a non-dimensional difference of radius decreased over about -0.2, despite decrease of the front rotor radius. That was because the outer parts of the rear rotor blades recovered the

wind velocity as the radial expansion of the fully developed stream tube of the front rotor became smaller than the radius of the rear rotor. Consequently, it was demonstrated that it was possible to optimize aerodynamic performanc of a counter-rotating wind turbine by using combinations of pitch angles, rotating speeds and radii of two rotors.

1   Jongh, J.A. de, Raghavan, K., Hulle, F.J.L. van, Power for the world -a common concept, study 1. ECN-C--95-037, ECN, 1996

2   World wind energy report 2009. WWEA (World Wind Energy Association), 2010

3   J. F. Manwell, J. G. McGowan and A. L. Rogers, *Wind energy explained, theory, desigh and application.* John Wiley & Sons Ltd., New York, 2002, pp.84-88

4   Erich Hau, *Wind turbines: fundamnetals, technologies, application, economics.* Springer, New York, 2000, pp.67-80

5   B. G. Newman, Actuator-disc theory for vertical-axis wind turbines. Journal of Wind Engineering and Industrial Aerodynamics, 15, 1983, pp.347-355

6   K. Appa, Energy innovations small grant (EISG) program (counter rotating wind turbine system). EISG Final Report, California, US, 2002

7   S. Jung, T. No and K. Ryu, Aerodynamic performance prediction of a 30kW counter-rotating wind turbine system. Renewable Energy, 30, 2005, pp.631-644

8   Toshiaki Kanemoto, and Ahmed Mohamed Galal, Development of intelligent wind turbine generator with tandem wind rotors and double rotational armatures. JSME International Journal, Series B, Vol. 49, No. 2, 2006, pp.450-457

9   W. Z. Shen, V. A. K. Zakkam, J. N. Sorensen and K. Appa, Analysis of counter-rotating wind turbines. Journal of Physics, Conference Series 75, 2007

10 Jang, T. J., and Heo, H. K., Study on the development of wind power system using mutually opposite rotation of dual rotors. Proceedings of the Renewable Energy 2008, 2008

11 S. Lee, H. Kim, and S. Lee, Analysis of aerodynamic characteristics on a counter-rotating wind turbine. Current Applied Physics, No. 10, 2010, pp.S339-S342

12 J. Gordon Leishman, *Principles of helicopter aerodynamics*, Cambridge University Press, New York, 2006, pp.727-747

13 Weick, F. E., *Aircraft propeller design*, McGraw-Hill Co., Inc., New York, 1930

14 Du Z, Selig M. A 3-D stall-delay model for horizontal axis wind turbine performance prediction. AIAA-98–0021, January, 1998

   Wilson, R. E., Lissaman, P. B. S. and Walker, S. N., *Aerodynamic performance of wind turbines*. Energy Research and Development Administration, ERDA/NSF/04014-73/1, 1976